Modeling 7

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1 Introduction to Linear Programming

Over the course of the semester we have explored algorithms to find the extrema of a function. Namely, a previous assignment utilized simulated annealing to solve the traveling salesman problem. Linear programming aims to achieve a similar goal, though through a much different process. While simulated annealing used the Boltzmann distribution to anneal into the solution state, linear programming utilizes topological spaces to find the local minimum of a (locally) linear function under a certain set of constraints. Generally, this system can be written as:

$$\text{Minimize} : c^T x$$

$$Ax \leq b$$

$$x \geq 0$$

Where x is the feasible vector, of values to be found, c is the vector of coefficients for the value to be minimized, A is the matrix of constraint on x and b is the vector representing the right hand side of the constraint equations. The values of x which gives the minimum value of $c^T x$ is called the optimum feasible vector. If these equations were plotted in the n-dimensional space the defining vectors existed in, some geometric properties could be observed. For every equality constraint on x, the dimension of the solution hyperpolygon, or simplex, would be reduced by one, and for every inequality constraint on x, there would be a hyperplane bounding it. It can then be surmised that the optimal feasible vector must exist on a vertex of the simplex. There are only two scenarios in which an optimal feasible vector doesn't exist. Either there are no feasible vectors at all, or the simplex is unbounded. The process of determining the minimum value of $c^T x$ is then simply a matter of constructing the simplex from the system of equations, finding an initial vector on a vertex, and the "walking" along the vertices of the simplex until a minimum is reached using the gradient. The initial vector is found by introducing slack variables to turn the inequality constraints into equality constraints. They are then used to find the first feasible vector. Fortunately, there are many off-the-shelf options for linear programming which will do the heavy lifting for us. For my code, I used `scipy.optimize.linprog`. The arguments for `scipy.optimize.linprog` are the column vector of coefficients for the value to be minimize (or maximized), the two dimensional matrix $A_u b$ for the left side of the inequalities, the column vector $B_u b$ for the right side of said equation, the respective matrices for the equality equations, and optional arguments like bounds on the values of x.

2 Diet

The applications of linear programming are far reaching, but some hit closer to home for college students than others. One such problem is your diet; how can you optimize your diet under a certain set of constraints? To achieve this, we'll take nutrition data from the USDA (provided by Andrej), and then optimize our diet with constraints on nutritional values. The values from the USDA were per 100 grams of food, so they were normalized so that all values were per 1 gram of food. That way, our solution values would be the amount of each food we should eat in grams. According to the assignment, the daily recommended daily nutritional intake is 70 g for fats, 310 g for carbs, 50 g for proteins, 1000 mg for calcium, and 18 mg for iron; thus, for all our diets, we will have at least that much of each nutrient. We also selected the 30 foods we consume the most from the USDA data file to narrow our food choices to ones we already commonly eat.

2.1 Minimizing Calories

One easy way to optimize our diet is to simply minimize our caloric intake; that is, we won't eat any more food than is absolutely necessary. It will also help reduce the intake of foods with "empty calories" such as sugar. Because we are selecting from 30 foods, the matrices are too large to put in here, but the matrix of coefficients (c) to be minimized was the calories per gram of each food, each row of $A_u b$ corresponds to a nutrients per gram of food, and the matrix $B_u b$ is the limiting values of each respective nutrient. There were no equality constraints. An additional constraint was the net weight of the food, which was limited to 2kg a day.

Food	**Amount (g)**
Avocados, raw, all commercial varieties	226.0
Bananas, raw	804.34
Spinach, raw	813.85
Peanut butter, smooth style, with salt	60.33
Syrups, maple	95.48

Table 1: Foods to minimize calories consumed (weight > 1 gram)

Perhaps the most surprising food in this group is the maple syrup. To the best of my knowledge this is because maple syrup contains a substantial number of carbs, so the program recognizes it as an efficient way to reach the required number of carbs. However, the other foods that appear are not overly surprising.

2.2 Minimizing fat

The dietitians among us will be quick to point out that calories aren't all bad; for example, strong man champion Brian Shaw consumes 12,000 calories daily. So instead of minimizing calories, what if we ensured we consumed at least 2000 calories and instead minimized fat?

Food	**Amount (g)**
Spinach, raw	625.87
Beverages, tea, green, brewed, regular	207.26
Syrups, maple	451.79
Yogurt, fruit variety, nonfat	715.08

Table 2: Foods to minimize fats consumed (weight $>$ 1 gram)

There are some unusual foods in here, but I believe there is justification for each. Spinach has vitamins and carbs, tea has vitamins, syrup has carbs, and nonfat yogurt has a decent amount of protein and no fat. This mixture results in only 4.14 grams of fat, just .2% of our daily food intake. While this may work for a few days, eventually we will die without more fat. So instead, what if we minimized fat while still getting the recommended 70g daily?

Food	Amount (g)
Cheese, parmesan, grated	24.19
Milk, whole, 3.25% milkfat, with added vitamin D	63.86
Egg, whole, raw, fresh	38.54
Milk, chocolate, fluid, commercial, reduced fat, with added calcium	95.45
Ice cream, soft serve, chocolate	40.56
Oil, olive, salad or cooking	9.87
Chicken, roasting, meat and skin, cooked, roasted	25.88
Turkey, breast, from whole bird, meat only, roasted	41.89
Ground turkey, cooked	29.77
Meatballs, frozen, Italian style	23.57
Apples, raw, with skin	77.08
Avocados, raw, all commercial varieties	35.53
Bananas, raw	81.15
Peaches, yellow, raw	74.03
Pears, raw	78.75
Plums, raw	74.65
Broccoli, raw	78.48
Carrots, raw	77.35
Spinach, raw	234.75
Nuts, cashew nuts, raw	15.39
Beef, grass-fed, strip steaks, lean only, raw	48.26
Beverages, tea, green, brewed, regular	68.38
Fish, salmon, pink, raw	40.08
Peanut butter, smooth style, with salt	11.63
Bread, wheat	90.06
Syrups, maple	123.9
Pasta, dry, enriched	79.4
PACE, Pico De Gallo	75.08
Tomato sauce, canned, no salt added	76.02
Yogurt, fruit variety, nonfat	97.94

Table 3: Foods to minimize fats consumed over 70g (weight $>$ 1 gram)

This results in a much greater variety of foods, and provides us with the necessary nutrients to survive. Our caloric intake is now up to 2338.6 per day, so this diet could be suitable for an athlete that wants to cut down on fat intake while maintaining an larger than average caloric intake.

2.3 Minimizing Calories with Variety

In the example where we minimized calories, we can see that our diet, while somewhat realistic, leaves something to be desired in the flavor department. So, to improve this, we can constrain our diet so that we don't consume more than a set amount of any one food — say 600g. This forces the algorithm to move away from the "ideal" model that contained only the best possible foods to an "imperfect" one that, while still meeting the constraints, is not minimized as well as the previous example. I've also increased the required caloric intake to 2500 calories and the daily weight to 3kg of food. The result is a highly varied, albeit less than convenient mixture of foods.

Food	**Amount (g)**
Cheese, parmesan, grated	32.18
Milk, whole, 3.25% milkfat, with added vitamin D	101.79
Egg, whole, raw, fresh	47.35
Milk, chocolate, fluid, commercial, reduced fat, with added calcium	140.7
Ice cream, soft serve, chocolate	53.01
Oil, olive, salad or cooking	4.06
Chicken, roasting, meat and skin, cooked, roasted	34.29
Turkey, breast, from whole bird, meat only, roasted	35.34
Ground turkey, cooked	35.69
Meatballs, frozen, Italian style	37.57
Apples, raw, with skin	116.98
Avocados, raw, all commercial varieties	61.87
Bananas, raw	93.8
Peaches, yellow, raw	126.06
Pears, raw	114.99
Plums, raw	117.49
Broccoli, raw	159.85
Carrots, raw	135.81
Spinach, raw	351.97
Nuts, cashew nuts, raw	26.72
Beef, grass-fed, strip steaks, lean only, raw	42.42
Beverages, tea, green, brewed, regular	244.24
Fish, salmon, pink, raw	39.36
Peanut butter, smooth style, with salt	20.74
Bread, wheat	60.2
Syrups, maple	68.46
Pasta, dry, enriched	46.23
PACE, Pico De Gallo	154.02
Tomato sauce, canned, no salt added	184.32
Yogurt, fruit variety, nonfat	88.71

Table 4: Foods that minimize calories and contrain intake for variety (weight>1g)

2.4 Price Minimization

Of course, perhaps the greatest factor for college students is the price of the food, not necessarily its nutritional value. My food amounts were all given in grams, so I was motivated to find a database that gave food prices by weight, not by the units they are sold in. I ended up using a database from the CNPP, an agency in the USDA. They had a food price database from 2003-2004[1], and while that data is more than a decade old (in fact it can drive), the relative prices of foods should have stayed almost the same, so the cheapest meal possible in 2004 is still the cheapest meal in 2020. What follows is a depressingly utilitarian meal that will cost you a grand total of $1.35 daily ($1.86 if you adjust for inflation).

Food	**Amount (g)**
Milk, chocolate, fluid, commercial, reduced fat, with added calcium	439.03
Peanut butter, smooth style, with salt	106.61
Pasta, dry, enriched	457.31

Table 5: Foods that minimize price of a daily food intake.

2.5 My Diet

Ultimately, none of these diets strike the right balance of foods I want to eat, so I instead implemented the following constraints (that differ from the daily recommended values): at least 2800 calories, 200 grams of protein, 150 grams of fat, no more than 600 grams of one food, and no more than 3kg of food. I also added the amount of sugar in all my foods, and decided to minimize my sugar intake. I initially tried to create my diet using the entire USDA file in the hopes I would discover some personal super food, but ultimately there were too many foods and the solutions often involved me eating about a gram of 3000 foods which is completely impractical. So, I limited it again to the 30 foods I eat the most. Here's the oddly Italian diet that I got.

Food	**Amount (g)**
Cheese, parmesan, grated	97.38
Oil, olive, salad or cooking	14.69
Turkey, breast, from whole bird, meat only, roasted	388.87
Nuts, cashew nuts, raw	32.82
Syrups, maple	42.05
Pasta, dry, enriched	376.99

Table 6: My ideal diet (weight > 1g)

3 Black-Scholes Model

The Black-Scholes model is a financial instrument which calculates the theoretical price of an option based on it's underlying asset's current price, volatility,

[1] https://www.fns.usda.gov/resource/cnpp-data

and the strike price and time to maturity of the option. It is widely used in the financial world and won its creators the Nobel Prize in Economics. In theory, with no transaction fees or latency, and with relatively small changes in price, it is possible to create a perfectly-hedged portfolio according to the model; however, it would need to be constantly recalculated, and markets never behave in such an idealized way. Nonetheless, we can create a perfectly hedged portfolio using linear programming. We want to maximize our profit, which can be calculated by subtracting the theoretical price from the market price. We can also hedge the portfolio by requiring that the sum of the respective "Greeks" is zero. The Greeks are variables in the Black-Scholes model that are calculated from the same variables as the theoretical price, and are present in the price expression. Unfortunately I was not able to get the code to work out for this because I did not use the right packages earlier on, so we're left with the disappointment that this assignment contained nothing but dietary problems, rather than a rich variety of interesting problems.